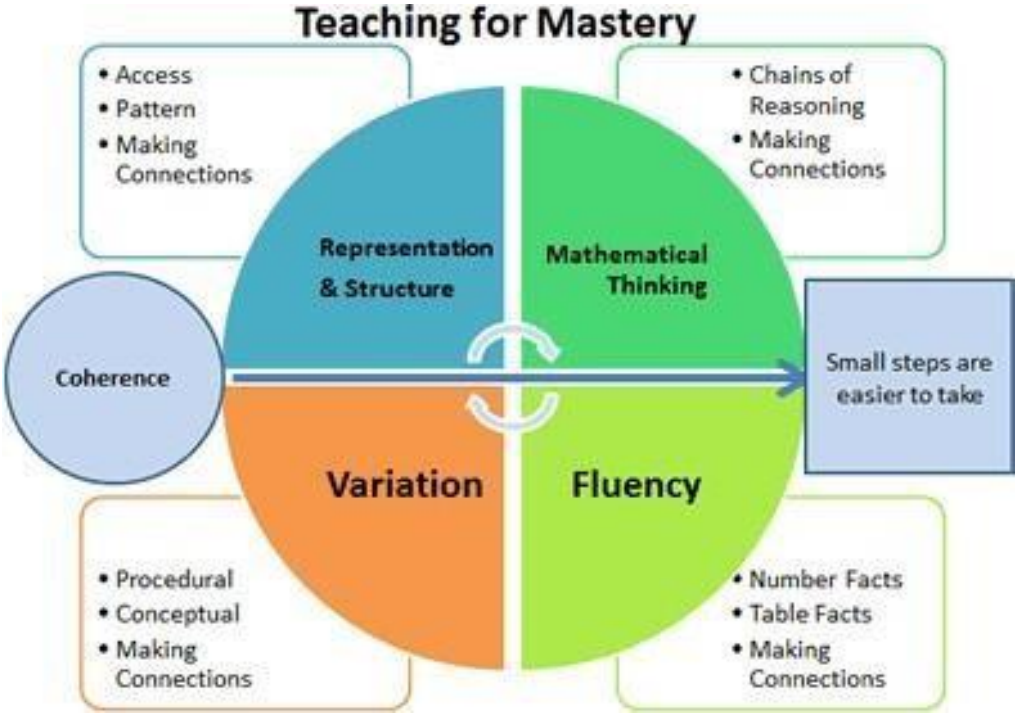
Maths Mastery & Calculations Policy



Concrete Pictorial Abstract approach

One of the key learning principles behind the Singapore maths textbooks is the concrete pictorial abstract approach, often referred to as the CPA approach. The concrete---pictorial--- abstract approach, based on research by psychologist Jerome Bruner, suggests that there are three steps (or representations) necessary for pupils to develop understanding of a concept. Reinforcement is achieved by going back and forth between these representations.

Concrete representation

The active stage --- a student is first introduced to an idea or a skill by acting it out with real objects. In division, for example, this might be done by separating apples into groups of red ones and green ones or by sharing 12 biscuits amongst 6 children. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

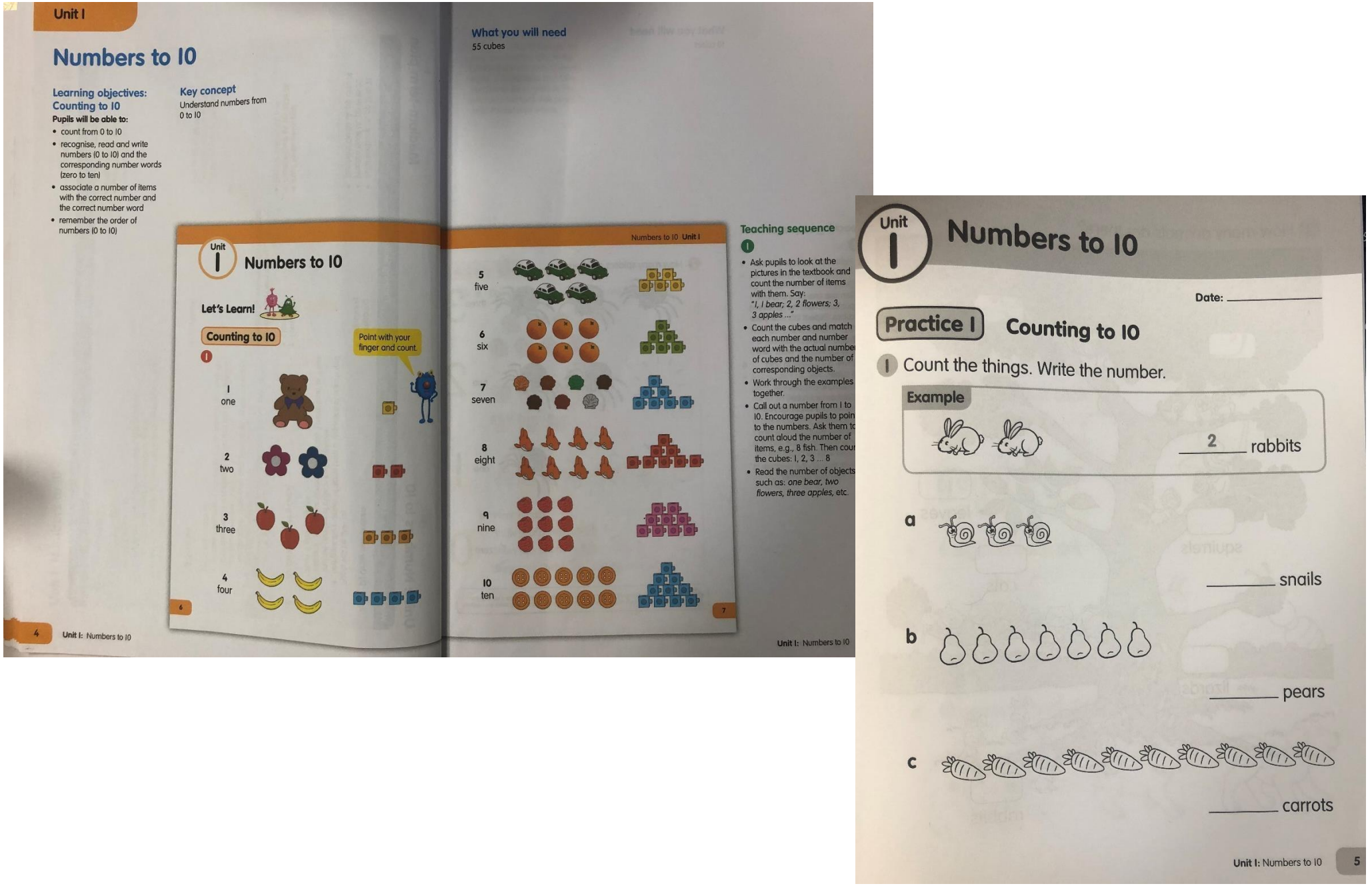
Pictorial representation

The iconic stage --- a student has sufficiently understood the hands---on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. In the case of a division exercise this could be the action of circling objects.

Abstract representation

The symbolic stage --- a student is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6 this is the ultimate mode, for it is clearly the most mysterious of the three.

Textbooks and practice books



Unit 1

Numbers to 10

Learning objectives: Counting to 10

Pupils will be able to:

- count from 0 to 10
- recognise, read and write numbers (0 to 10) and the corresponding number words (zero to ten)
- associate a number of items with the correct number and the correct number word
- remember the order of numbers (0 to 10)

Key concept

Understand numbers from 0 to 10

What you will need

55 cubes

Unit 1 Numbers to 10

Let's Learn!

Counting to 10

Point with your finger and count.

1 one

2 two

3 three

4 four

5 five

6 six

7 seven

8 eight

9 nine

10 ten

Numbers to 10 Unit 1

Teaching sequence

- Ask pupils to look at the pictures in the textbook and count the number of items with them. Say: *"1, 1 bear; 2, 2 flowers; 3, 3 apples ..."*
- Count the cubes and match each number and number word with the actual number of cubes and the number of corresponding objects.
- Work through the examples together.
- Call out a number from 1 to 10. Encourage pupils to point to the numbers. Ask them to count aloud the number of items, e.g., 8 fish. Then count the cubes: 1, 2, 3 ... 8
- Read the number of objects such as: *one bear, two flowers, three apples*, etc.

4 Unit 1: Numbers to 10

6

7

Unit 1: Numbers to 10

Unit 1 Numbers to 10

Date: _____

Practice 1 Counting to 10

1 Count the things. Write the number.

Example

2 rabbits

a _____ snails

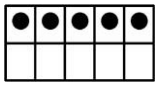
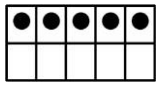
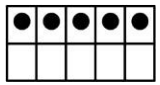
b _____ pears

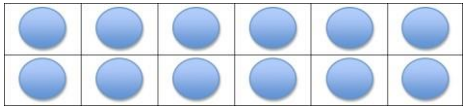
c _____ carrots

Unit 1: Numbers to 10 5

Concrete materials

Progression in the use of manipulatives to support learning

Foundation	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Real---life objects	Real---life objects	Real---life objects	Real---life objects	Real---life objects	Real---life objects	Real---life objects
0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards	0 – 9 digit cards
Number track to 10	Number line to 20	Number line to 100	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
Numbered counting stick	Counting stick	Counting stick	Counting stick	Counting stick	Counting stick	Counting stick
Tens frame 	Tens frame 	Tens frame 				
	Place value charts – Tens and ones	Place value charts – Hundreds, tens and ones	Place value charts – Thousands, hundreds, tens & ones	Place value charts – Ten thousands, thousands, hundreds, tens, ones & tenths	Place value charts to a million and three decimal places	Place value charts to 10 million and three decimal places
Interlocking cubes --- Use one colour to represent one amount	Interlocking cubes --- Use one colour to represent one amount	Dienes	Dienes	Dienes	Dienes	Dienes
			Place value counters	Place value counters	Place value counters	Place value counters
	Place value arrow cards – tens and ones	Place value arrow cards – tens and ones	Place value arrow cards – H, T, O	Place value arrow cards – Th, H, T, O	Place value arrow cards	Place value arrow cards
Part---part---whole mat	Part---part---whole mat	Part---part---whole mat	Part---part---whole model	Part---part---whole model	Part---part---whole model	Part---part---whole model
Bar model with real-life objects	Bar model with real life objects/pictorial objects/representative objects eg. counters	Bar model with counters /Dienes progressing to numbers	Bar model with numbers	Bar model with numbers	Bar model with numbers	Bar model with numbers
Bead strings – ten	Bead strings - twenty	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred
Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes
			Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods
Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters
Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount

Maths Working Wall		
Build it!	Use a real---life representation of the concept which children can see, touch and feel.	
Draw it!	Show a pictorial representation of the concept.	
Solve it!	Show the mathematical representation of the concept.	6 x 2 = 12 2 x 6 = 12 12 ÷ 2 = 6 12 ÷ 6 = 2 Factors of 12 are: 1, 2, 3, 4, 6 and 12
Practise it!	Encourage children to practice the concept. ***Interactive opportunity*** – ask children to respond to questions, encourage them to add what they know, leave homework for children to take to master the concept.	1 x 2 = 2 2 x 2 = 4 3 x 2 = 6 etc.
Challenge it!	Set a challenge to be solved. ***Interactive opportunity*** – leave real---life objects or manipulatives for children to use to help solve the challenge.	How many different ways can 12 eggs be arranged into arrays? What if you try 24 eggs?
Say it!	Use vocabulary related to the concept	Multiply, times, repeated addition, array, divide, group, multiples, factors

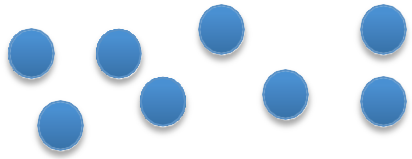
Classroom Visual Prompts

Foundation	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Big focus 10	Big focus 20	Big focus 100				
Numicon number line with Numicon shapes	Numicon number line with Numicon shapes	Numicon number line	Fractions number line	Fractions and decimals number line	Fractions, decimals and percentages number line	Fractions, decimals and percentages number line
	Odd and even numbers	Odd and even numbers			Prime, square and cube numbers	Prime, square and cube numbers
	Number pairs totalling 10 Number pairs totalling 20	Number pairs totalling 10 Multiples of 10 totalling 100	Number pairs totalling 10 Multiples of 10 totalling 100			
0 – 10 number line / track	0 ---20 number line	0 – 100 number line	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
	100 square	100 square	100 square	100 square		
Number names from 0 - 10	Number names from 0 - 20	Number names from 0 – 100	Number names from 0 - 1000	Number names to one million	Number names to one trillion	Number names to one trillion
Real coins Large coins	Real coins Large coins	Real coins Large coins	Real coins Large coins	Real coins Large coins	Real coins Large coins	Real coins Large coins
	1, 2, 5 and 10 times tables	2, 3, 4, 5 and 10 times tables	All times tables up to 12 x 12	All times tables up to 12 x 12	All times tables up to 12 x 12	All times tables up to 12 x 12
			Roman numerals	Roman numerals	Roman numerals	Roman numerals
		< , > and = signs	< , > and = signs	<, > and = signs	<, > and = signs	<, > and = signs
Real---life / pictorial fractions	Real---life / pictorial fractions	Fractions including fraction number line/wall	Fractions including fraction number line/wall	Fractions including fraction number line/wall	Fractions, decimals and percentages including fraction number line/wall	Fractions, decimals and percentages including fraction number line/wall
						BODMAS
2d and 3d shapes	2d and 3d shapes	2d and 3d shapes	2d and 3d shapes	2d and 3d shapes	2d and 3d shapes	2d and 3d shapes

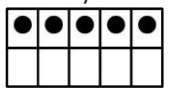
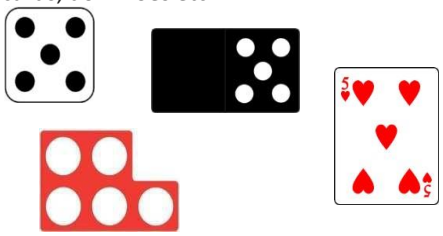
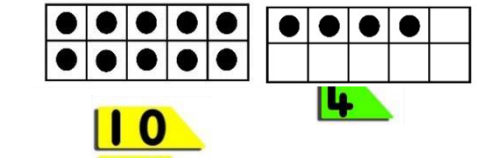
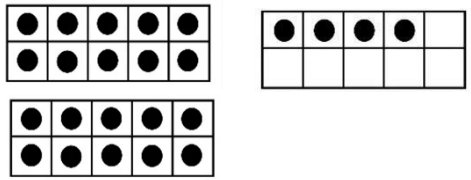
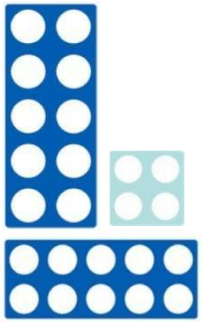
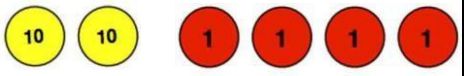
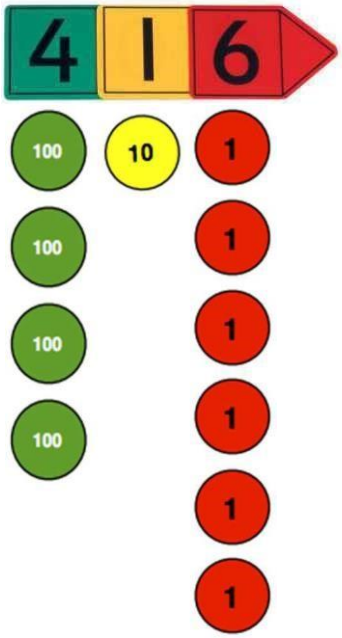
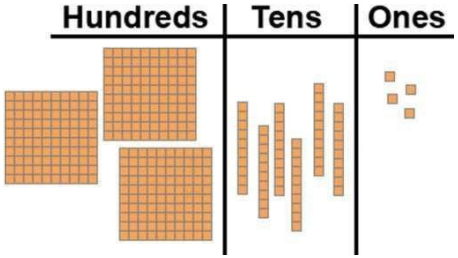
Journaling in maths books--- recording our work

Thinking caps	What do they mean?
 Explain It!	Descriptive journal- describes method of solution You choose- write the method to solve the problem that you think is most effective I choose- write and explain the solution using the method we chose together as a class from the structured discussion.
 Convince me!	Convince a friend/ teacher that you're right
 Prove it!	Can you use another method to show that your answer is correct?
 Use it!	This time children need to use the method to work out another problem. Does it work?
 Evaluative it!	Look at all of the methods we could use to solve the problem, which works best for you? Give reasons why it works best for you (evaluate the different methods)
 Tell me a story	Creative journal- story about doughnuts can you write your own story- problem posing, students write the problem based on an equation e.g. doughnuts
 Investigate it!	Present an open ended question with a number of solutions, such as I have 14 doughnuts, I give away 8 I have 6 doughnuts, could I have started with a different number and still have 6 left? e.g. 16-10=6, what if the number I'm giving away is not more than 10? e.g. 15-9=6 what would leave me with 6 doughnuts, but I don't want to give away more than 10.

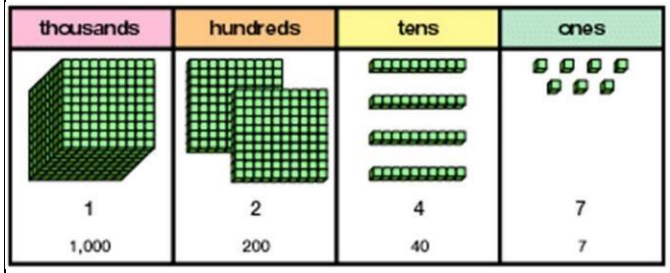
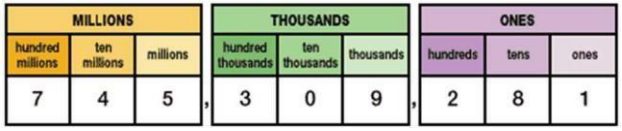
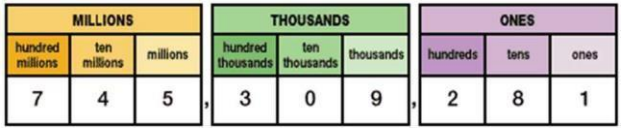
Progression in the teaching of counting in the foundation stage

Pre-counting The key focus in pre-counting is an understanding of the concepts more, less and the same and an appreciation of how these are related. Children at this stage develop these concepts by comparison and no counting is involved.	**Ordering** Count by reciting the number names in order forwards and backwards from any starting point.	**One to one correspondence** One number word has to be matched to each and every object. Lack of coordination is a source of potential error – it helps if children move the objects as they count, use large rhythmic movements, or clap as they count.	**Cardinality (Knowing the final number counted is the total number of objects)** Count out a number of objects from a larger collection. Know the number they stop counting at will give the total number of objects.
Pre-counting ideas *Provide children with opportunities to sort groups of objects explicitly using the language of **more** and **less.*** *Which group of apples has the most? Which group of apples has the least?*	**Ordering ideas** *Provide children with opportunities to count orally on a daily basis. Rote count so that children are able to understand number order and can hear the rhythm and pattern. Use a drum or clap to keep the beat.*	**One to one correspondence ideas** *Play counting games together moving along a track, play games involving amounts such as knocking down skittles.* *Use traditional counting songs throughout the day ensuring children have the visual/kinaesthetic resources eg. 5 little ducks, 10 green bottles*	**Cardinal counting ideas** ***How many bananas are in my fruit bowl? Allow children to physically handle the fruit.*** *Provide children with objects to point to and move as they count and say the numbers.*
Subitising (recognise small numbers without counting them) Children need to recognise small amounts without counting them eg. dot patterns on dice, dots on tens frames, dominoes and playing cards as well as small groups of randomly arranged shapes stuck on cards.	**Abstraction** You can count anything – visible objects, hidden objects, imaginary objects, sounds etc. Children find it harder to count things they cannot move (because the objects are fixed), touch (they are at a distance), see, that move around. Children also find it difficult to count a mix of different objects, or similar objects of very different sizes	**Conservation of number – MASTERY!** Ultimately children need to realise that when objects are rearranged the number of them stays the same.	**End of year counting expectations** • count reliably to 20 • count reliably up to 10 everyday objects • estimate a number of objects then check by counting • use ordinal numbers in context eg first, second, third • count in twos, fives and tens • order numbers 1-20 • say 1 more/1 less than a given number to 20
Subitising ideas *Provide children with opportunities to count by recognising amounts.*	**Abstraction ideas** *How many pigs are in this picture?* *Provide children with a variety of objects to count*	**Conservation of number** The amount is 8 and it doesn't change 	

Progression in the teaching of place value

Foundation	Year 1	Year 2	Year 3 onwards
Understanding ten	**Understanding numbers up to 20**	**Understanding numbers up to one hundred**	**Understanding numbers up to one thousand**
A TENS FRAME is a simple maths tool that helps children: • Keep track of counting • See number relationships • Learn addition to 10 • Understand place value Use ***tens frames*** flash cards daily to ensure children recognise amounts. Use empty ***tens frames*** to fill with counters to enable children to understand number relationships. Either fill the ***tens frame*** in pairs or in rows. In rows shows 5 as a benchmark. Children can easily see more than 5 or less.  Setting the counters in pairs, naturally allows the children to see addition concepts. Include other visual images such as dice, cards, dominoes etc. 	'Ten' is the building block of our Base 10 numeration system. Young children can usually 'read' two---digit numbers long before they understand the effect the placement of each digit has on its numerical value. A child might be able to correctly read 62 as sixty--- two and 26 as twenty---six, and even know which number is larger, without understanding why the numbers are of differing values. Ten---frames can provide a first step into understanding two---digit numbers simply by the introduction of a second frame. Placing the second frame to the right of the first frame, and later introducing numeral cards, will further assist the development of place--- value understanding.   	Continue developing place value through the use of ***tens frames***.     	Continue developing place value through the use of manipulatives.  Use Dienes blocks and place value charts 

Progression in the teaching of place value

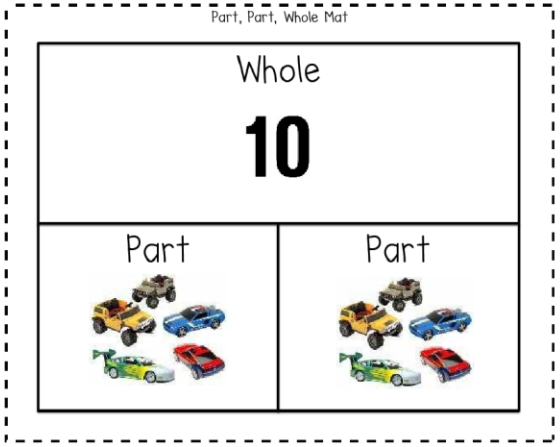
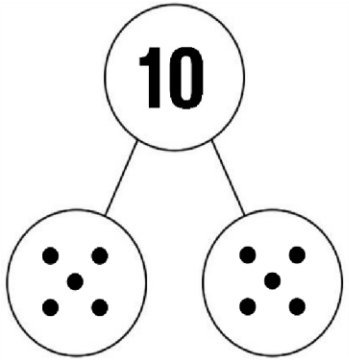
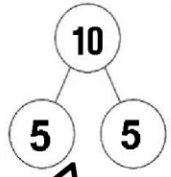
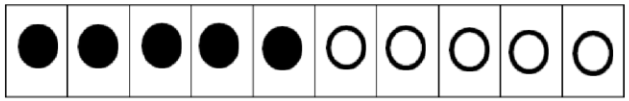
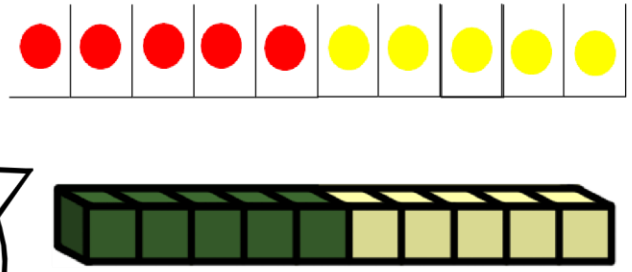
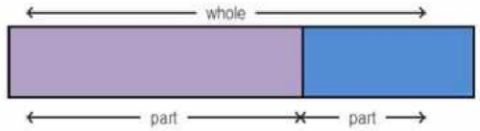
Year 4	Year 5	Year 6
Understanding numbers up to ten thousand	**Understanding numbers up to one million including decimals**	**Understanding numbers beyond one million including decimals**
Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters • Dienes blocks • Place value charts  	Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters (including decimal counters) • Dienes blocks • Place value charts  	Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters (including decimals counters) • Dienes blocks • Place value charts 

Progression in the teaching of calculations

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number & counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method- regrouping. (up to 3 digits)	Column method--- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- -regrouping. (Decimals- with different amounts of decimal places).
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

Progression in the teaching of calculations

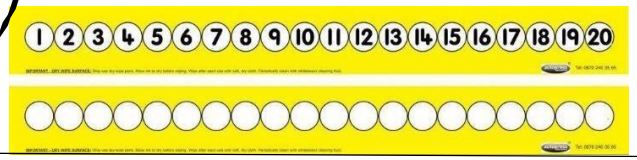
ADD IT!

Objective and strategies	Concrete BUILD IT/USE IT!	Pictorial DRAW IT!	Abstract SOLVE IT!
Combine two parts to make a whole model. Part–part–whole model			
Teach the children that the cubes/counters represent the real-life objects.			Use the part–part whole diagram as shown above to move into the abstract.
Use cubes to add two numbers together as a group or in a bar.		 Part + Part = Whole Whole – Part = Part	5 + 5 = 10 10 = 5 + 5 Use the term ‘number sentence’.

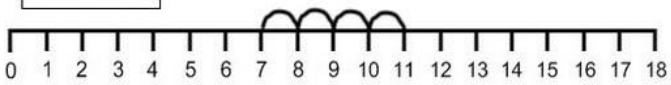
Start at the larger number and count on

Start with the larger number on the bead string then count on 1 by 1 to find the answer.

Use counters on a number track to count on.



7 + 4 = 11



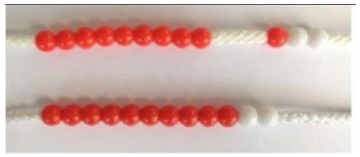
Start at the larger number on the number line and count on in ones or in one jump to find the answer.

4 + 7 = 11

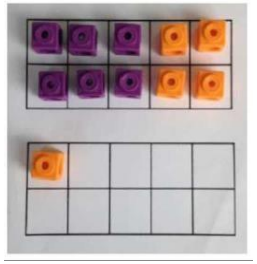
Place the larger number in your head and count on the smaller number to find your answer.

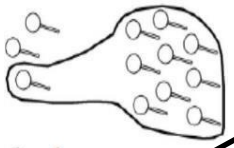
Regrouping to make 10.

Start with the bigger number and use the smaller number to make 10.



6 + 5 = 11

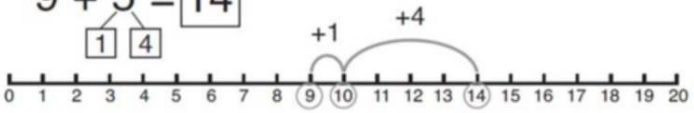




3 + 9 =

Use pictures or a number line. Regroup or partition the smaller number to make 10.

9 + 5 = 14



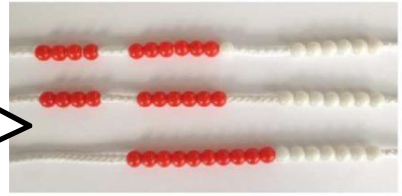
If I am at seven, how many more do I need to make 10. How many more do I add on now?

Adding three single digits.

Encourage children to use known facts.

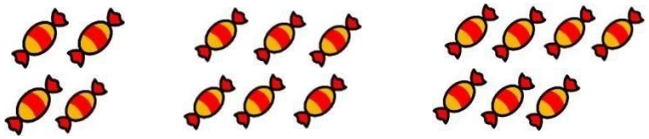
4 + 7 + 6= 17

Put 4 and 6 together to make 10. Add on 7.

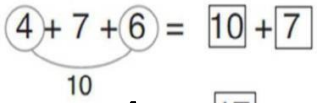


Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.

Add together three groups of objects. Draw a picture to recombine the groups to make 10.



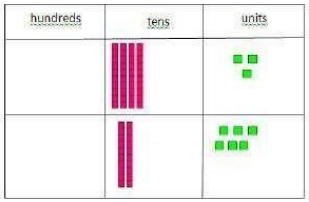
4 + 6 + 7 = 17



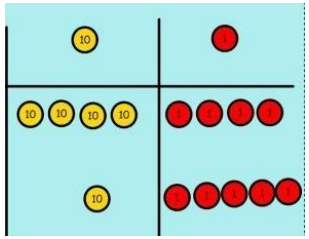
Combine the two numbers that make 10 and then add on the remainder.

Column method– no regrouping

Use Dienes to add tens and ones before moving on to place value counters.



$$\begin{array}{r} 43 \\ +\ 26 \\ \hline \\ \hline \end{array}$$



After practically using the base 10 blocks and place value counters, children can draw the Dienes to help them to solve addition calculations.

hundreds	tens	ones
	////	□ □ □
	//	□ □ □ □ □ □
	6	9

After practically using Dienes, children can draw the 'tens' and 'ones'.

Calculations

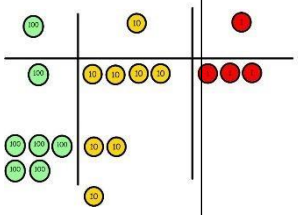
21 + 42 =

$$\begin{array}{r} 21 \\ +\ 42 \\ \hline \end{array}$$

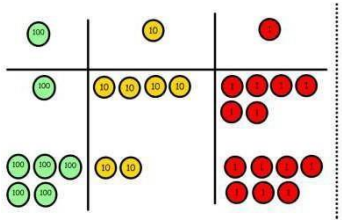
Only select numbers which do not involve regrouping.

Column method– regrouping

Make both numbers on a place value grid.



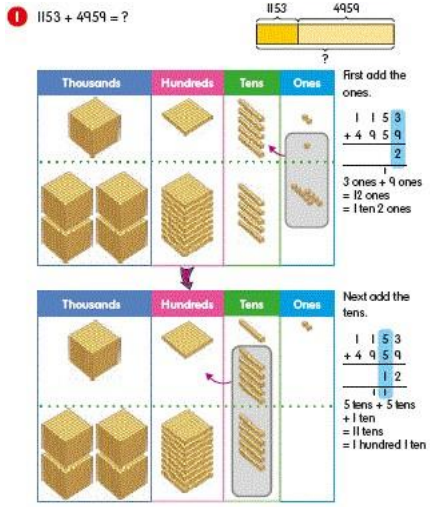
146
+ 527



$$\begin{array}{r} 146 \\ +\ 527 \\ \hline \end{array}$$

Add up the units and exchange 1o ones for one 10 and so on.

Continue using place value counters as children begin to work with decimals.



If necessary children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding.

hundreds	tens	ones
/	////	□ □ □ □ □ □
/////	//	□ □ □ □ □ □ □
6	6	3
	1	

$$\begin{array}{r} 72.8 \\ +\ 54.6 \\ \hline 127.4 \\ 1\ 1 \end{array}$$

$$\begin{array}{rrcrrr} 2 & 3 & . & 3 & 6 & 1 \\ & 9 & . & 0 & 8 & \mathbf{0} \\ 5 & 9 & . & 7 & 7 & \mathbf{0} \\ + & 1 & . & 3 & \mathbf{0} & \mathbf{0} \\ \hline 9 & 3 & . & 5 & 1 & 1 \\ 2 & 1 & & 2 & & \end{array}$$

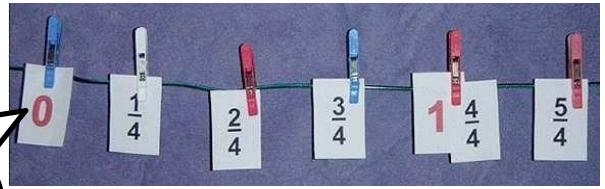
$$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ 1\ 1 \end{array}$$

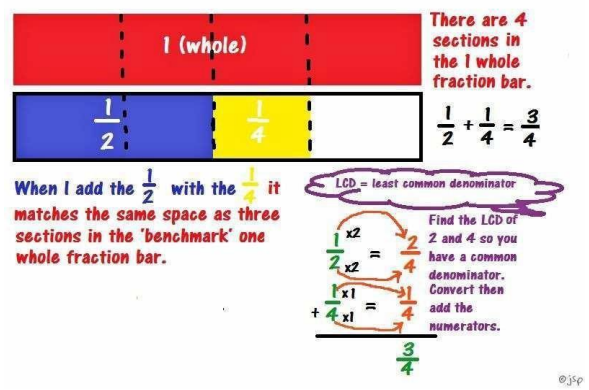
As the children move on, introduce decimals with the same number of decimal places.

Then move onto decimals with a different number of decimal places.

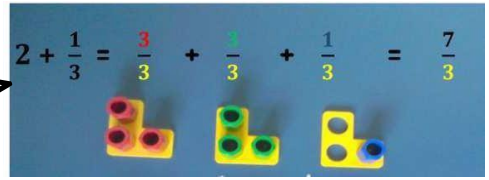
Add fractions

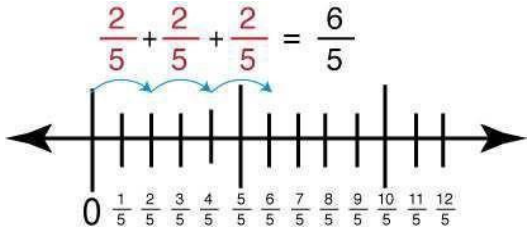
Count in fraction steps using real objects and a number line.



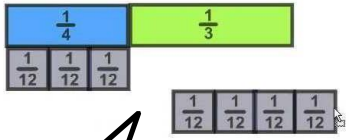


Use Numicon to add fractions.





$$\frac{1}{4} + \frac{1}{3}$$



Use the bar model to add fractions.

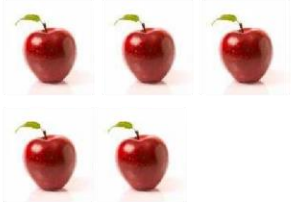
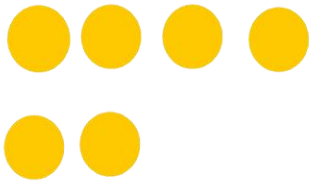
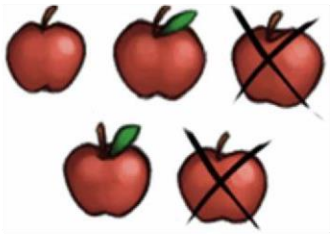
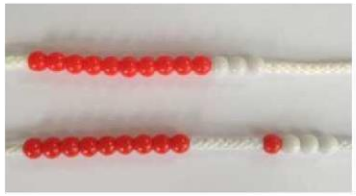
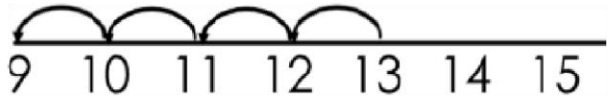
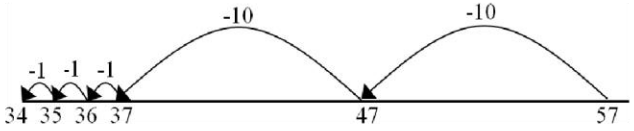
$$\frac{1}{4} + \frac{1}{3} =$$

$$\frac{1 \times 3}{4 \times 3} + \frac{1 \times 4}{3 \times 4}$$

$$\frac{3}{12} + \frac{4}{12} = \frac{7}{12}$$

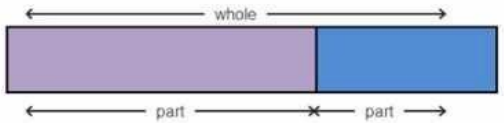
Progression in Calculations Policy

SUBTRACT IT!

Objective and strategies	Concrete BUILD IT/USE IT!	Pictorial DRAW IT!	Abstract SOLVE IT!
Taking away ones	Use real-life physical objects, counters, cubes etc. to show how objects can be taken away.  6 – 2 = 4 	Cross out drawn objects to show what has been taken away.  5 – 2 = 3	4 = 6 – 2 18 --- 3= 15 8 – 2 = 6
Counting back Use counters and move them away from the group whilst counting backwards.	Make the larger number in the subtraction calculation. Move the beads along the bead string whilst counting backwards in ones. 13 – 4  	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line. 	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. Children will need regular practice counting backwards.

Use cubes to subtract a number from the bar.

Use the bar



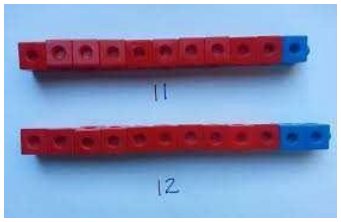
Part + Part = Whole

Whole – Part = Part

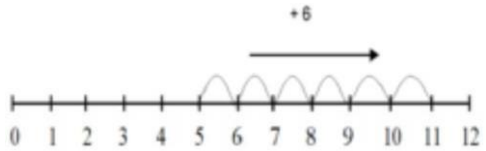
Find the difference

Compare amounts and objects to find the difference.

Use cubes to build towers or make bars to find the difference.

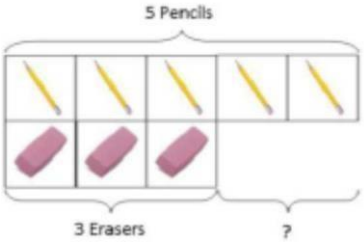


Count on to find the difference.



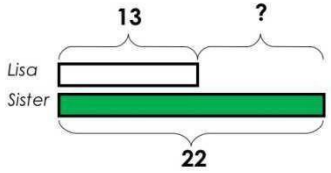
Hannah has 23 pencils, Helen has 15 pencils. Find the difference between the number of pencils.

Use basic bar models with items to find the difference.



Comparison Bar Models

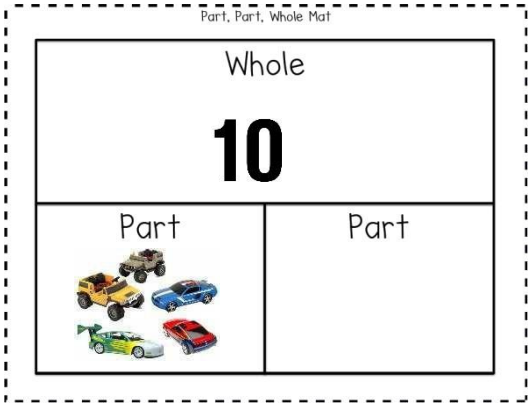
Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.



Draw bars to find the difference between two numbers.

Part-part-whole model

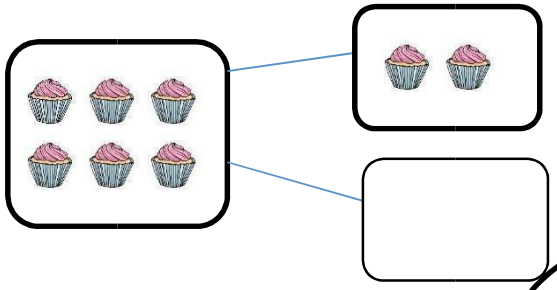
Link to addition- use the part whole model to help explain the inverse.

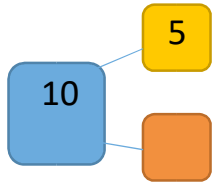


If 10 is the whole and 5 is one of the parts. What is the other part?

10 – 5 = or 10 –-? =5

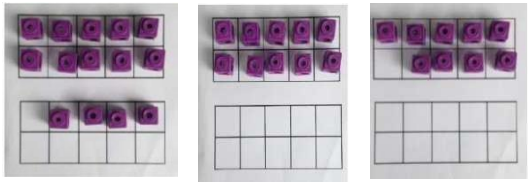
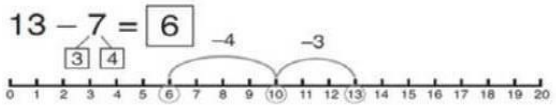
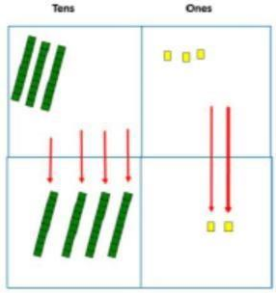
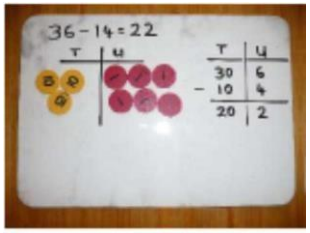
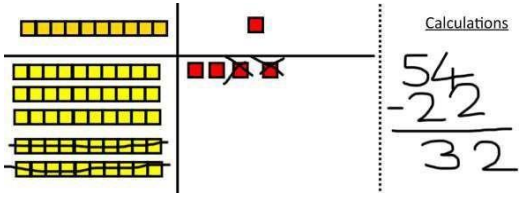
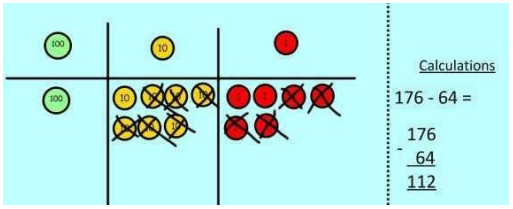
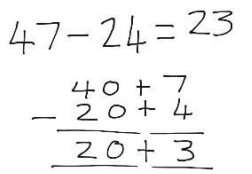
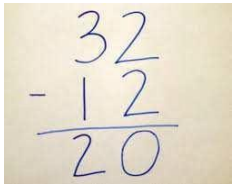
Use a pictorial representation of objects to show the part-part-whole model.

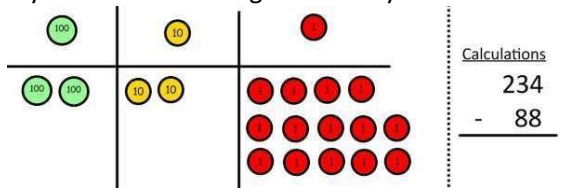
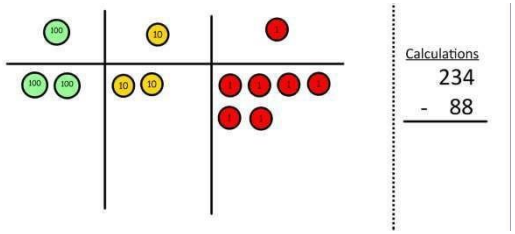
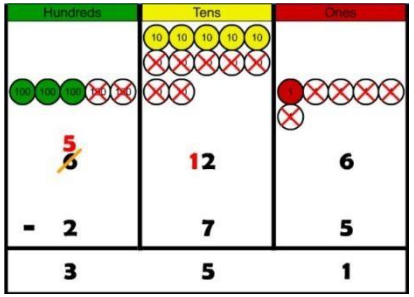
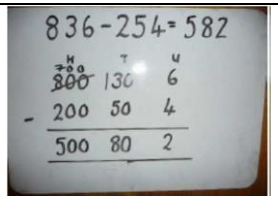
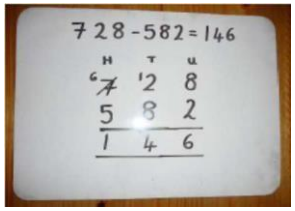




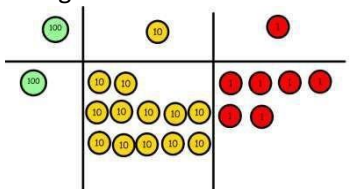
10 –-5 = 5 or 5 = 10 – ?

Move to using numbers with the part-part-whole model.

Make 10	14 – 5 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8 = How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping	75--- 42 = Use Dienes to make the bigger number then take the smaller number away.  Show how you partition numbers to subtract. Again make the larger number first.  	Draw the Dienes or place value counters alongside the written calculation to help to show working.   	This will lead to a clear written column subtraction.  

Column method with regrouping	Make the larger number with the Dienes or place value counters. Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Calculations 234 - 88 Now I can subtract my ones.  Calculations 234 - 88	Draw the counters onto a place value grid and show what has been taken away by crossing the counters out as well as clearly showing the exchanges made.  When confident, children can find their own way to record the exchange/regrouping.	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens



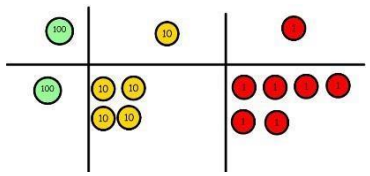
Calculations

234
- 88

Now I can take away eight tens and complete my subtraction

including decimals.

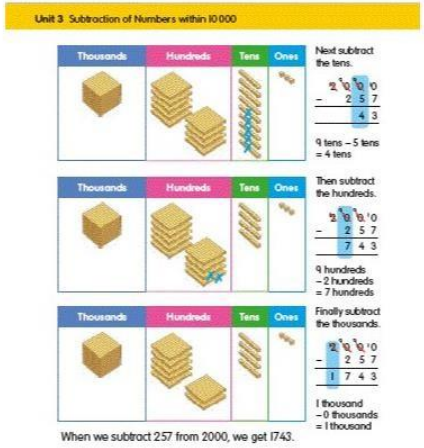
$$\begin{array}{r} {\scriptstyle 5\ \ 12\ \ \ \ 1} \\ 2\ \not{6}\ \not{3}\ .\ \mathbf{0} \\ -\quad 2\ 6\ .\ 5 \\ \hline 2\ 3\ 6\ .\ 5 \end{array}$$



Calculations

$$\begin{array}{r} 134 \\ -\ 88 \\ \hline 146 \end{array}$$

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.



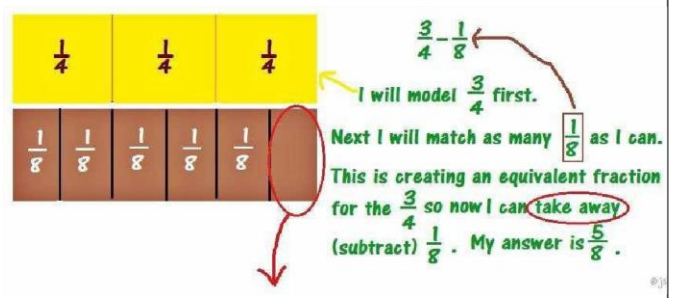
Subtract fractions

If there are five fifths and I eat one fifth, what fraction of the cake is left?

The cake has been divided into five slices. Each part is one fifth of the whole cake.

Draw a bar model to represent the cake.

Progress onto subtracting fractions with different denomiators.



$\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$

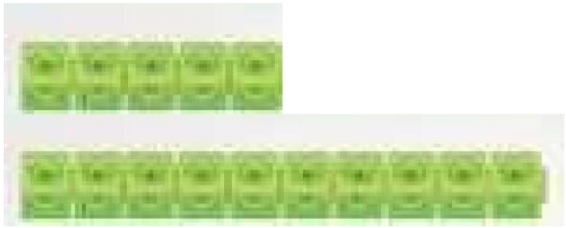
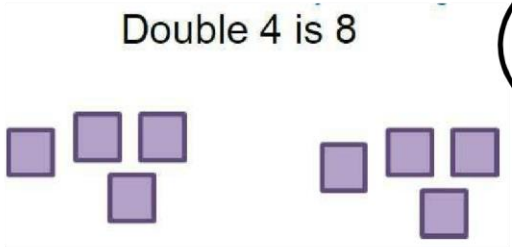
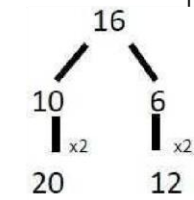
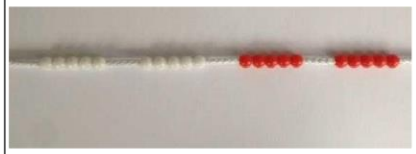
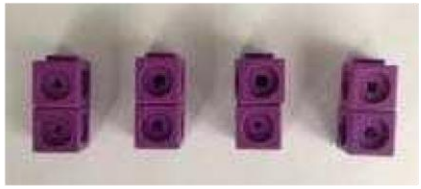
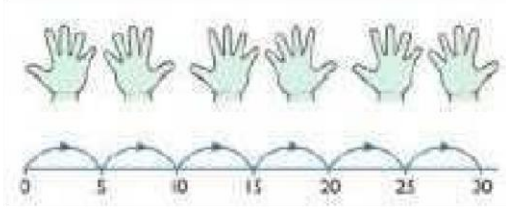
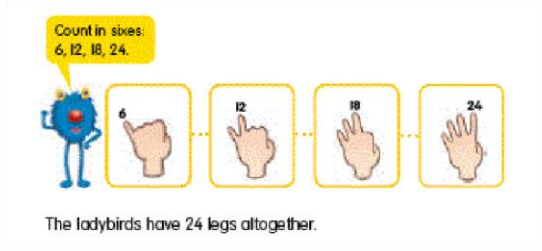
$\frac{3}{4} - \frac{1}{8} =$

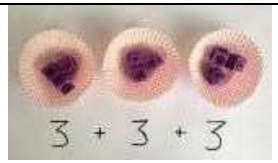
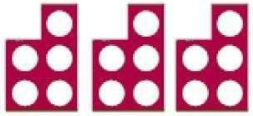
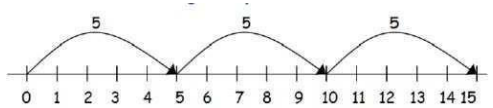
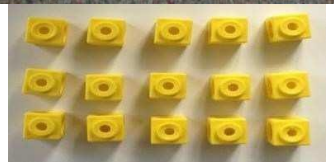
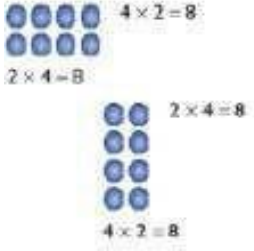
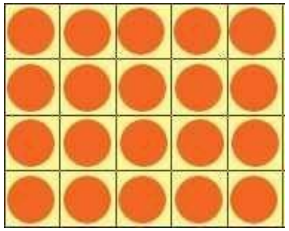
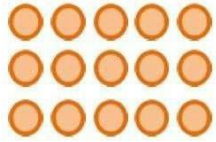
$\frac{3x2}{4x2} - \frac{1}{8}$

$\frac{6}{8} - \frac{1}{8} = \frac{5}{8}$

Progression in Calculations Policy

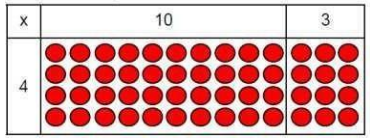
MULTIPLY IT!

Objective and strategies	Concrete BUILD IT/USE IT!	Pictorial DRAW IT!	Abstract SOLVE IT!
Doubling Double five is ten.	Use practical activities to show how to double a number.  5 x 2 = 10	Draw pictures to show how to double a number. 	Double 16 Double the 10 then double the 6.  Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10
Arrays--- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

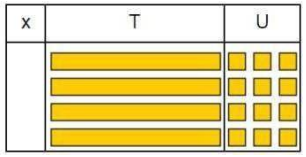
Grid Method

Show the link with arrays to first introduce the grid method.



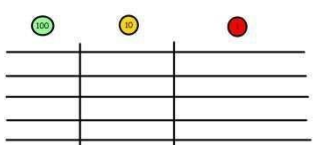
4 rows of 10 4 rows of 3

Use Dienes to move towards a more compact method.



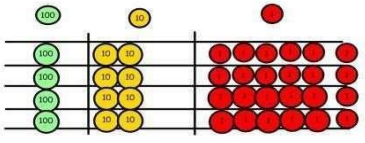
4 rows of 13

Use place value counters to show finding groups of a number eg. multiplying by 4 so we need 4 rows.



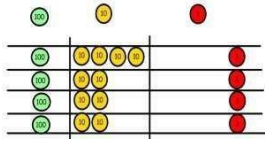
Calculations
4 x 126

Fill each row with 126.



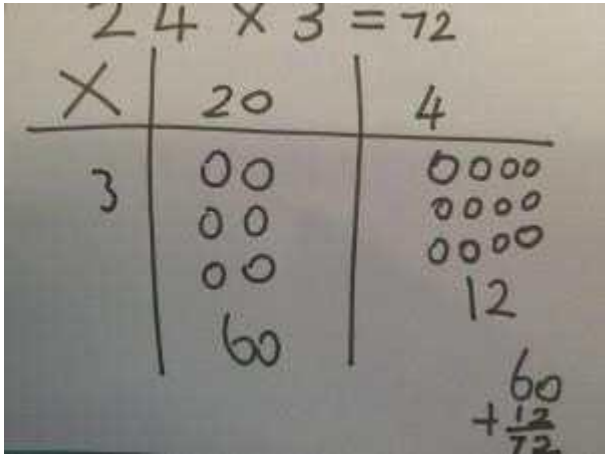
Calculations
4 x 126

Add up each column, starting with the ones making any exchanges needed.



Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

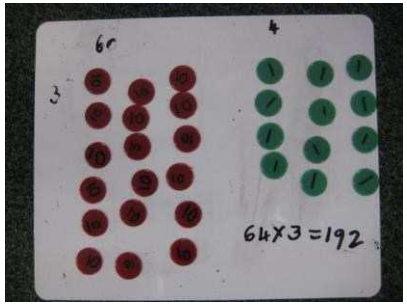
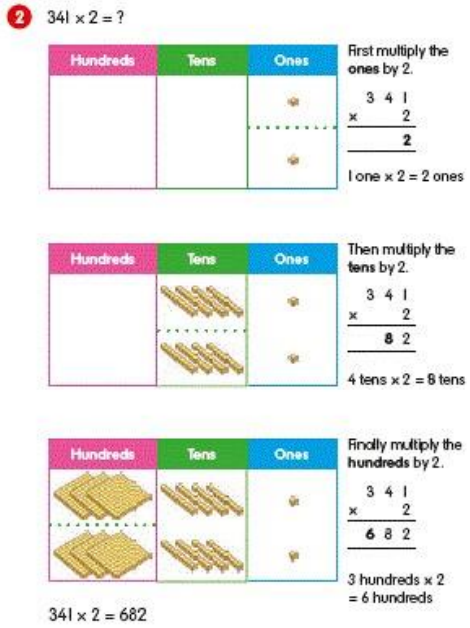
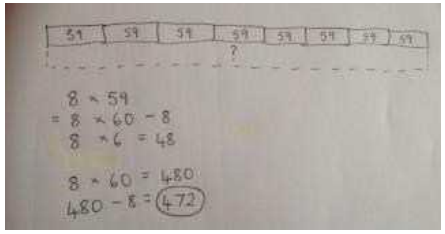
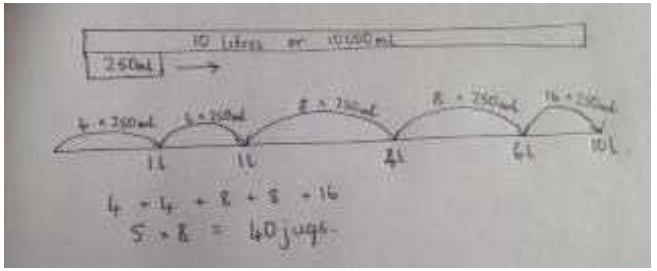
×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

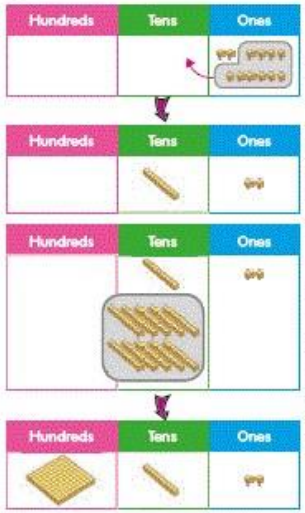
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.  	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768 74 x 63 12 210 240 + 4200 4662 This moves to the more compact method. 327 x 53 981 ← 327 x 3 16350 ← 327 x 50 17331

Let's Learn!

Multiplication with regrouping in ones, tens, hundreds and thousands

1 $656 \times 2 = ?$



First multiply the **ones** by 2.

$$\begin{array}{r} 6\ 5\ 6 \\ \times\quad 2 \\ \hline 2 \end{array}$$

6 ones × 2 = 12 ones

Regroup the ones:
12 ones = 1 ten 2 ones

Then multiply the **tens** by 2.

$$\begin{array}{r} 6\ 5\ 6 \\ \times\quad 2 \\ \hline 1\ 2 \end{array}$$

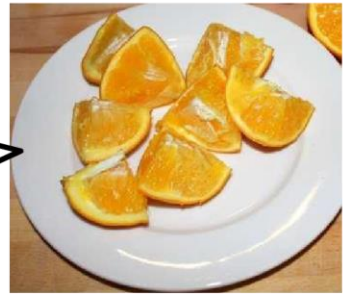
5 tens × 2 = 10 tens

Add the tens:
10 tens + 1 ten = 11 tens

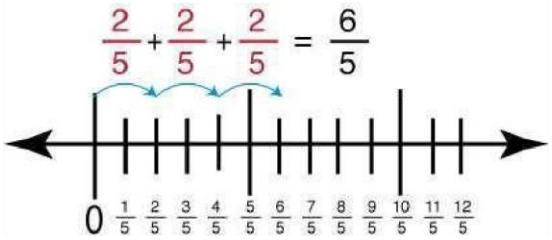
Regroup the tens:
11 tens = 1 hundred 1 ten

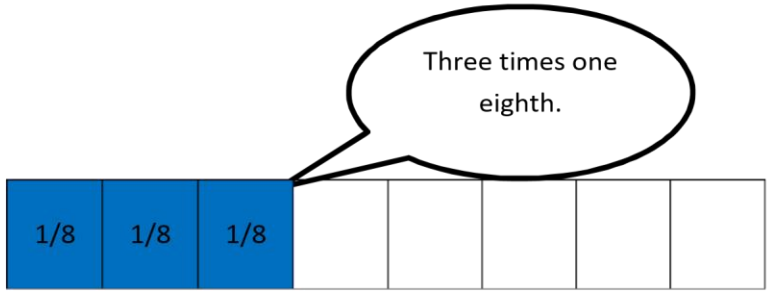
Multiplication of fractions

What would three lots of one eighth be?



Count in fraction steps (repeated addition)





$^{10}/_{8} = 1\ ^{2}/_{8}$
$^{9}/_{8} = 1\ ^{1}/_{8}$
$^{8}/_{8} = 1$
$^{7}/_{8}$
$^{6}/_{8}$
$^{5}/_{8}$
$^{4}/_{8} = {}^{1}/_{2}$
$^{3}/_{8}$
$^{2}/_{8} = {}^{1}/_{4}$
$^{1}/_{8}$

$3 \times {}^{1}/_{8} =$

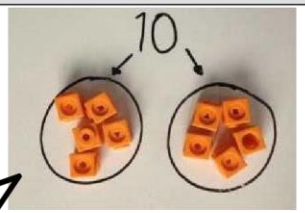
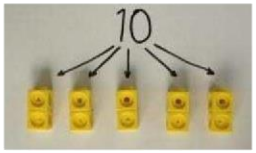
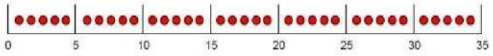
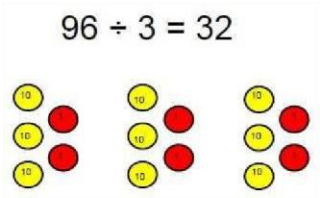
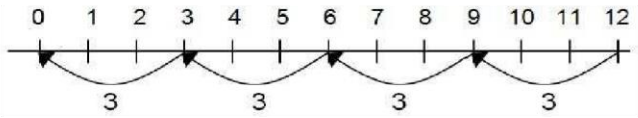
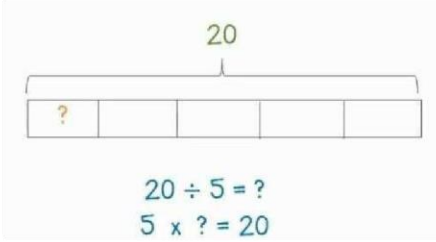
$\frac{3}{1} \times \frac{1}{8} = \frac{3}{8}$

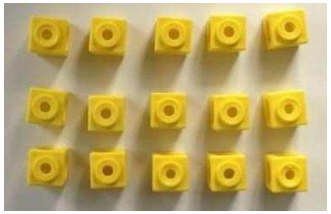
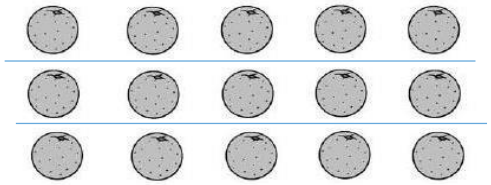
Multiply the numerators together then multiply the denominators.

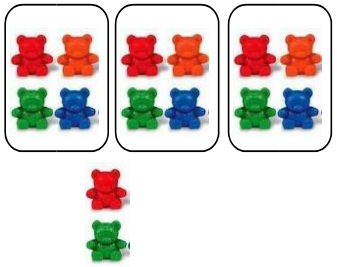
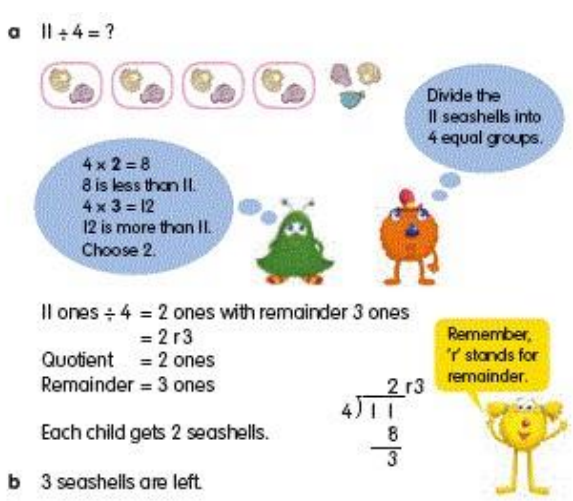
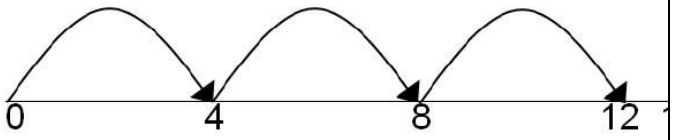
Progression in Calculations Policy

DIVIDE IT!

It is important to make links with fractions

Objective and strategies	Concrete BUILD IT/USE IT!	Pictorial DRAW IT!	Abstract SOLVE IT!
Sharing objects into groups If we are dividing by two we are finding one half.	 I have 10 cubes; can you share them equally into 2 groups?	Children use pictures or shapes to share quantities. 8 ÷ 2 = 4	One half of 14 is 7 ½ of 14 = 7 14 ÷ 2 = 7 Share 9 cakes between three people. 9 ÷ 3 = 3
Division as grouping If we are dividing by three we are finding one third.	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	~~28 ÷ 7 = 4~~ Divide 28 into 7 groups. How many are in each group?

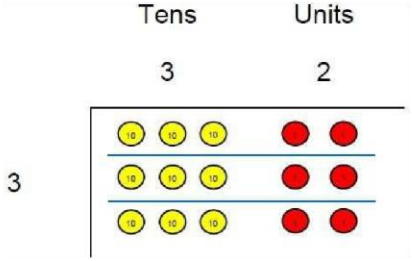
Division within arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over   	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  	Complete written divisions and show the remainder using r. 

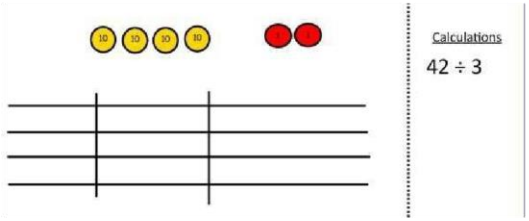
Short division

Find one third of 96.

96 ÷ 3 =

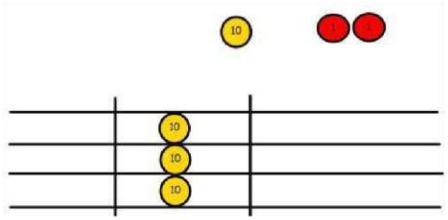


Use place value counters to divide using the bus stop method alongside

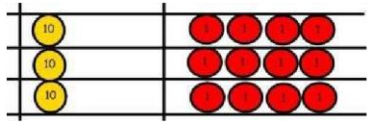


42 ÷ 3=

Start with the biggest place value; share 40 into three groups. Put 1 ten in each group then 1 ten left over.

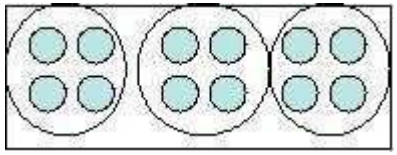


Exchange this ten for ten ones and then share the ones equally among the groups.



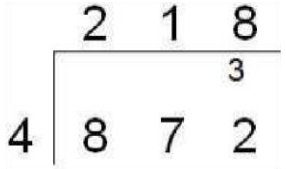
Look how much is in 1 group so the answer is 14.

Students can continue to use drawn diagrams with

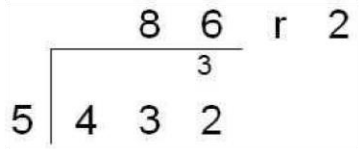


dots or circles to help them divide numbers into equal

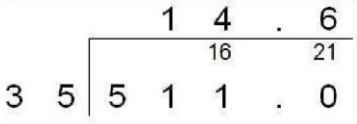
Begin with divisions that divide equally with no remainder.



Move onto divisions with a remainder.



Finally move into decimal places to divide the total accurately.



groups.

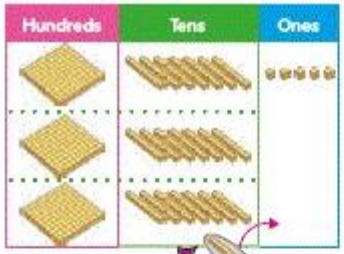
Encourage them to move towards counting in multiples to divide more efficiently.

	 Then divide the **ones** by 3. 3 ones ÷ 3 = 1 one So 63 ÷ 3 = 21. Each child gets 21 twigs. $3\overline{)63}$ → 21; 6; 3; 3; 0		

Long division

(chunking method)

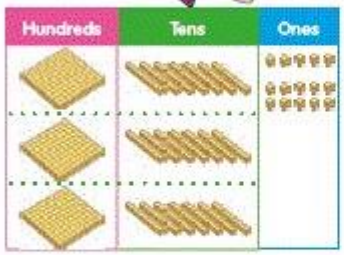
Divide by single digit then progress to dividing by two digit number



Then divide the **tens** by 3.

22 tens ÷ 3
= 7 tens with remainder 1 ten

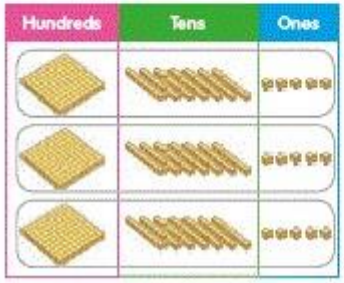
```
  17
3)525
  3
  22
  21
   1
```



Regroup the remainder ten:
1 ten = 10 ones

Add the ones:
10 ones + 5 ones = 15 ones

```
  17
3)525
  3
  22
  21
   15
```



Finally divide the **ones** by 3.

15 ones ÷ 3 = 5 ones

```
  175
3)525
  3
  22
  21
   15
   15
    0
```

So 525 ÷ 3 = 175.

Each restaurant receives 175 cabbages.

```
   86 r2
5 | 432
    200   (40 x 5)
    232
    200   (40 x 5)
     32
     30   (6 x 5 )
      2
```

```
13 | 1937
-  1300   13 x 100
    637
-   520   13 x 40
    117
-   117   13 x 9
      0
```

Ask pupils to work in pairs. Ask them to find the missing numbers for the following:

```
  □□□
3)528
  3
  2□
  2□
  □□
  □□
   □
```

Division of fractions Half of the pizza divided into three equal parts.	½ ÷ 3 = 	½ ÷ 3 =  Half of the bar divided into three equal parts.	½ ÷ 3 = $\frac{1}{2} \div \frac{3}{1} =$ $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Times Table Policy

TIMES IT!

Times Tables are at the heart of mental arithmetic, which in itself helps form the basis of a child's understanding and ability when working with number. Once the children have learnt their times tables by heart, they are then able to work far more confidently and efficiently through a wide range of more advanced calculations.

At Beever Primary School, we believe that through a variety of interactive, visual, engaging and rote learning techniques, most children can achieve the full times table knowledge by the time they enter Year 5.

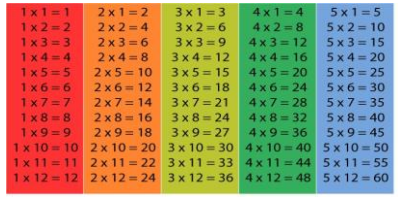
Reception	Year 1	Year 2	Year 3	Year 4	Year 5 and 6
I can count in steps of 1	I can count in steps of 5	I know my 5 times table	I know my 6 times table	I know my 11 times table	Regular consolidation of all times tables
I can count in steps of 2	I know my 1 times table	I know my 3 times table	I know my 7 times table	I know my 12 times table	
I can count in steps of 10	I know my 2 times table	I know my 4 times table	I know my 8 times table		
I can count in steps of 5	I know my 10 times table		I know my 9 times table		

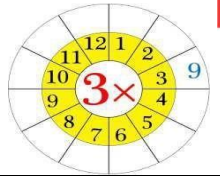
Rote learning

Times tables will be recited daily.

Chant as: 'One times two is two, two times two is four, three times two is six ….'

Also chant as 'one multiplied by two is two, once two is two, one lot of two is two, one group of two is two, the product of one and two is two etc.'





Display

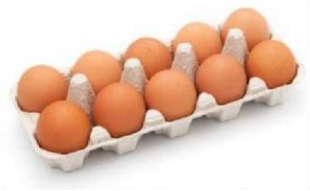
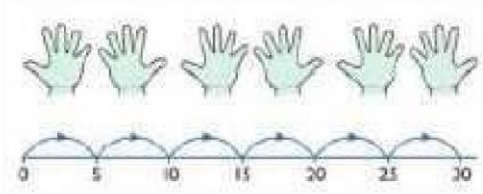
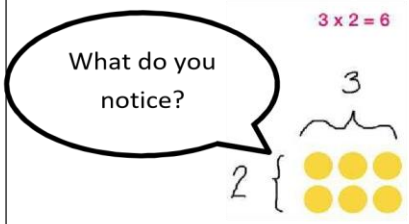
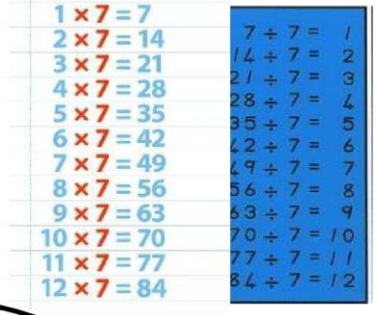
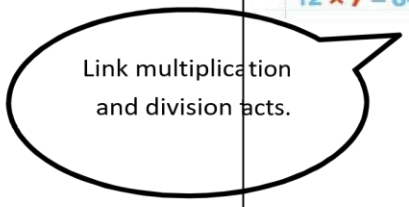
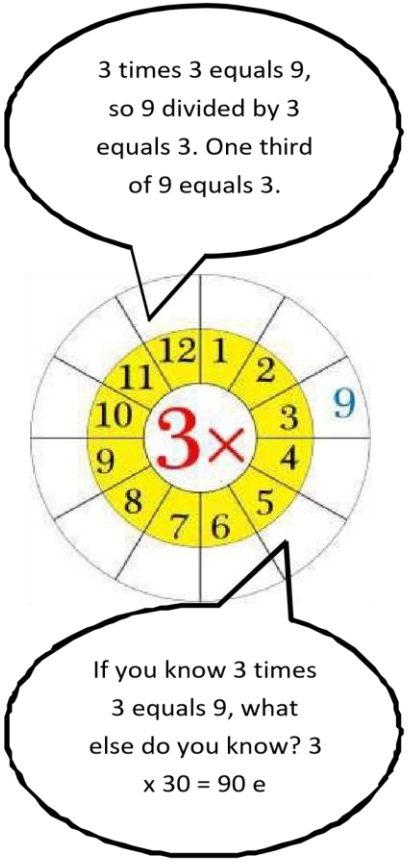
Times tables should be on display at the front of all classrooms, for children to use as support and reference.
Year 1: 1, 2, 5 and 10 times tables should be displayed.
Year 2: 1, 2, 3, 4, 5 and 10 times tables should be displayed

KS2: All times tables up to 12 x 12 should be available for children. The display must be large enough for all children to see and on table top resources where necessary. Individual times tables should be displayed.

Homework

Children need to be sent home times table homework on a regular basis. This can be in the form of times table 'challenges', identifying times table patterns, practicing with parents or listening to Times Tables songs on Mathletics.

Process of teaching times tables

Children will be taught the concept of multiplication using practical resources.	Children will progress on to using number lines or pictures.	Children will count in multiple steps.	Children will recite times tables by rote. Links will be made with 'grouping' and division whilst times tables are being taught.
Concrete **BUILD IT! / USE IT!**	**Pictorial** **DRAW IT!**	**Abstract stage 1** **SOLVE IT!**	**Abstract stage 2** **PRACTISE IT!**
Count in multiples supported by concrete objects in equal groups.  Use real-life arrays or build arrays.  	 Use a number line or pictures to continue support in counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30 Record multiplication number sentences.   	Recite times tables by rote orally. 

TENS FRAME IDEAS

LIFE SIZE TEN FRAME	Create a life---size ten frame in the classroom and outdoor play area. Use counters, pennies, teddies, gingerbread men, children etc.
FLASH	Flash ***ten frame*** briefly and have children write the number on a whiteboard. Using whiteboards, rather than having children say the number, ensures that all children attempt to respond and allows the teacher to assess class progress. When the response is oral, not all child responses are audible. Encourage children to share the different strategies used to find the total number of dots for cards, "How did you see it?" This can be varied by asking children to write the number and draw the pattern they saw, or by having them build the number flashed on their own blank frame.
FLASH: ONE MORE	Once children are familiar with the basic patterns, and know them automatically, flash a 10 frame or dot card and ask them to name the number that is one more than the number flashed. Variation: ask children to give the number that is two more/one less/double/ten more than the number flashed.
I WISH I HAD TEN	Flash a dot card or ten frame showing 9 or less and say, "I wish I had 10". Children respond with the part that is needed to make ten. The game can focus on a single whole, or the "wish I had" number can change each time. Variation: teacher flashes card and children write the complement of ten on individual whiteboards with dry erase markers.
I WISH I HAD 12	As above but children respond with how many more are needed to make twelve. Children should be confident in facts of 10 before this is attempted. For example to go from 8 to 12, they should realise they need 2 more to get to 10, then 2 more to 12. 2 and 2 is 4. Variation: Children draw an empty number line on their whiteboards to show the two jumps used to get to the target number.
1 MORE **1 LESS** **10 MORE** **10 LESS**	The following four prompts are written on the board: one more one less ten more ten less The teacher flashes a dot or ten frame card as the 'starting number'. The first child selects one prompt. For example, if the teacher flashes a card showing '5' the first child might say, "one more than 5 is 6", the second child might say, "ten more than 6 is 16", and the third child might say, "one less than 16 is 15". Continue until all children have had a turn.
TEEN FRAME FLASH (11---20)	**Teen Frame Flash (11---20)** Once children are subitizing ***ten frame*** patterns 0--- 10, cards showing larger numbers (i.e. more than one ten frame) should be introduced. Use mental math sessions with the following key questions: How many? How many more than 10? As children become familiar with the 'teen' patterns introduce further questions to develop number relationships. • What is one more/two more than the number I flashed? • What is one less/two less than the number I flashed? • How far away is the number I flashed from twenty? • Double the number I flash. • What is the near Doubles fact? (i.e., if 15 is flashed, children answer 7+8)
MULTIPLES	Flash a ***tens frame*** and ask children to give you the product if the number you flash was multiplied by 2, 5 etc.